Published by Institute of Physics Publishing for SISSA/ISAS
Received: *April 1, 2003*
Accepted: *May 9, 2003*

Non-gaussian features of primordial fluctuations in single field inflationary models

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Abstract: We compute the three point correlation functions for primordial scalar and tensor fluctuations in single field inflationary models. We obtain explicit expressions in the slow roll limit where the answer is given terms of the two usual slow roll parameters. In a particular limit the three point functions are determined completely by the tilt of the spectrum of the two point functions. We also make some remarks on the relation of this computation to dS/CFT and AdS/CFT. We emphasize that (A)dS/CFT can be viewed as a statement about the wavefunction of the universe.

Keywords: Physics of the Early Universe, Large Scale Structure Formations, Cosmology of Theories beyond the SM.

http://jhep.sissa.it/archive/papers/jhep052003013/jhep052003013.pdf

JHEP05(2003)013

Contents

1. Introduction and summary of results

Single field inflationary models predict to a good approximation a gaussian spectrum of primordial fluctuations. The size of non-gaussian corrections is expected to be small and was estimated in [1]–[4].

In this paper we will compute the correction to the gaussian answer to leading order in the slow roll parameters but with the precise numerical coefficient as well as momentum dependence. In single field inflationary models one considers the action of a single scalar field coupled to gravity. This action is expanded around a spatially homogeneous solution. The leading order term in the expansion is quadratic in the small fluctuations around the homogeneous answer. Since a free field is a collection of harmonic oscillators and these harmonic oscillators start their life in the ground state one finds that the fluctuations are gaussian to leading order. The non-gaussian effects come from the cubic interaction terms in the full action. These interaction terms arise from the non-linearities of the Einstein action as well as from non-linearities in the potential for a scalar field. We compute the cubic terms in the lagrangian. These cubic terms lead to a change both in the ground state of the quantum field as well as non-linearities in the evolution. These two effects can be computed in a simple way by following the usual rules of quantum field theory, assuming the standard choice of vacuum for an interacting field.

We parameterize the scalar fluctuations in terms of ζ which is the gauge invariant variable that remains constant outside the horizon [5]. We schematically denote by γ the tensor (or gravity wave) fluctuations. In the slow roll approximation we obtain

$$\begin{aligned}
\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\right\rangle &= \frac{H^4}{M_{pl}^4}\frac{1}{\epsilon}\delta^3\left(\sum \vec{k}_i\right)\mathcal{M}_1 \\
\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}h_{\vec{k}_3}\right\rangle &= \frac{H^4}{M_{pl}^4}\frac{1}{\epsilon}\delta^3\left(\sum \vec{k}_i\right)\mathcal{M}_2 \\
\left\langle \zeta_{\vec{k}_1}\gamma_{\vec{k}_2}\gamma_{\vec{k}_3}\right\rangle &= \frac{H^4}{M_{pl}^4}\delta^3\left(\sum \vec{k}_i\right)\mathcal{M}_3 \\
\left\langle \gamma_{\vec{k}_1}\gamma_{\vec{k}_2}\gamma_{\vec{k}_3}\right\rangle &= \frac{H^4}{M_{pl}^4}\delta^3\left(\sum \vec{k}_i\right)\mathcal{M}_4
\end{aligned} \tag{1.1}$$

where ϵ is a slow roll parameter and $\mathcal{M}_i$ are homogeneous functions of the momenta of degree k^{-6} whose explicit form we give below. The dependence on H/M_{pl} is due to the fact that we are looking at the cubic term in the action. Of course, the power of k^{-6} in $\mathcal{M}_i$ comes from approximate scale invariance.

In the limit that one of the momenta in (1.1) is much smaller than the other two there is a simple argument which determines the three point functions. This simple argument can also be used to understand the factors of slow roll parameters in (1.1). The argument is the following. Consider the limit $k_1 \ll k_{2,3}$. Suppose the small momentum corresponds to a scalar fluctuation ζ. The fluctuation ζ_{k_1} is frozen by the time the other two momenta cross the horizon. So its only effect is to rescale the other two momenta so that we get a contribution proportional to the violation in scale invariance of the two point function of the two fluctuations with large momenta. In other words the first and third line of (1.1) are proportional to the tilt of the scalar and tensor fluctuations respectively (times the product of the corresponding two point functions). More explicitly, for the three ζ correlator we get[1]

$$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle \sim -\langle \zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}\rangle k\frac{d}{dk}\langle \zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = -n_s\langle \zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle\langle \zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}\rangle\,, \qquad k_1 \ll k_2, k_3 \tag{1.2}$$

where n_s is the tilt of the scalar spectrum defined by $\langle\zeta\zeta\rangle \sim k^{-3+n_s}$ so that n_s is the deviation from scale invariance.

In order to understand the behavior when k_i are all of the same order of magnitude we need to do the computation by expanding the action. Then the answer is a more complicated function of k_i but the size of the correlation function does not numerically change much. The other two correlation functions in (1.1) can also be understood in the limit that one of the k_i is very small through a similar simple argument which we give in section 4.

[1]We are dropping some factors of $(2\pi)^3\delta(\sum \vec{k})$. These are more explicitly written later.

Another way of presenting the argument is as follows. Since the wavefunction of gravity in a space that is approximately de-Sitter is supposed to have the properties of a conformal field theory [6, 7], the three point functions that we computed above can be related to correlation functions of the stress tensor in the hypothetical dual CFT. In the limit that one of the k_i is much smaller than the other two the form of the three point function is determined in terms of the two point function by the following argument. If one of the k_i is very small we can approximate it by zero, so that the corresponding insertion of the trace of the stress tensor represents the effects of an infinitesimal rescaling of coordinates. So this three point function is determined by how the two point function behaves as we rescale the coordinates. This is why the three point function $\langle \zeta^3 \rangle$ is equal to the tilt of the spectrum of the two point function in the regime $k_1 \ll k_{2,3}$.

Komatsu and Spergel have performed an analysis of the detectability of non-gaussian features of the temperature fluctuations [8, 9]. Their analysis was made for an expected signal which had a slightly different k dependence from the one in $\mathcal{M}_1$ above. This probably would not change their answer too much. Ignoring this point, one would conclude from their analysis that this level of non-gaussianity is not detectable from CMB measurements alone. A more explicit discussion is given below. In some models with more than one field non-gaussianity can be large [10].

Finally we point out that these computations can also be used in investigations of AdS/CFT and dS/CFT. These dualities can be viewed as a statement about the wave-function of the universe. We relate explicitly the computation of stress tensor correlators in the dS and AdS case. They are related by a simple analytic continuation. We also clarify the relation between stress tensor correlators and the spectrum of fluctuations of metric perturbations.

This paper is organized as follows. In section two we review the standard results that follow from the quadratic approximation and give the gaussian answer. In section three we expand the action to third order. In section four we compute the three point functions. In section five we make some remarks on the relationship of these computations to the dS/CFT and AdS/CFT correspondences.

2. Review of the quadratic computation

The computation of primordial fluctuations that arise in inflationary models was first discussed in [11, 12, 13, 14, 5, 15] and was nicely reviewed in [16].

The starting point is the lagrangian of gravity and a scalar field which has the general form

$$S = \frac{1}{2} \int \sqrt{g} \left[R - (\nabla \phi)^2 - 2V(\phi) \right] \tag{2.1}$$

up to field redefinitions. We have set $M_{pl}^{-2} \equiv 8\pi G_N = 1$,[2] the dependence on G_N is easily reintroduced.

[2]Note that this definition of M_{pl} is different from the definition that some other authors use (including Planck).

The homogeneous solution has the form

$$ds^2 = -dt^2 + e^{2\rho(t)}dx_i dx_i = e^{2\rho}(-d\eta^2 + dx_i dx_i) \tag{2.2}$$

where η is conformal time. The scalar field is a function of time only. ρ and ϕ obey the equations

$$\begin{aligned} 3\dot{\rho}^2 &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ \ddot{\rho} &= -\frac{1}{2}\dot{\phi}^2 \\ 0 &= \ddot{\phi} + 3\dot{\rho}\dot{\phi} + V'(\phi)\,. \end{aligned} \tag{2.3}$$

The Hubble parameter is $H \equiv \dot{\rho}$. The third equation follows from the first two. We will make frequent use of these equations.

If the slow roll parameters are small we will have a period of accelerated expansion. The slow roll parameters are defined as

$$\begin{aligned} \epsilon &\equiv \frac{1}{2}\left(\frac{M_{pl}V'}{V}\right)^2 \sim \frac{1}{2}\frac{\dot{\phi}^2}{\dot{\rho}^2}\frac{1}{M_{pl}} \\ \eta &\equiv \frac{M_{pl}^2 V''}{V} \sim -\frac{\ddot{\phi}}{\dot{\rho}\dot{\phi}} + \frac{1}{2}\frac{\dot{\phi}^2}{\dot{\rho}^2}\frac{1}{M_{pl}} \end{aligned} \tag{2.4}$$

where the approximate relations hold when the slow roll parameters are small.

We now consider small fluctuations around the solution (2.3. We expect to have three physical propagating degrees of freedom, two from gravity and one from the scalar field. The scalar field mixes with other components of the metric which are also scalars under SO(2) (the little group that leaves $\vec{k}$ fixed). There are four scalar modes of the metric which are δg_{00}, δg_{ii}, $\delta g_{0i} \sim \partial_i B$ and $\delta g_{ij} \sim \partial_i\partial_j H$ where B and H are arbitrary functions. Together with a small fluctuation, $\delta\phi$, in the scalar field these total five scalar modes. The action (2.1) has gauge invariances coming from reparametrization invariance. These can be linearized for small fluctuations. The scalar modes are acted upon by two gauge invariances, time reparametrizations and spatial reparametrizations of the form $x^i \to x^i + \epsilon^i(t,x)$ with $\epsilon^i = \partial_i \epsilon$. Other coordinate transformations act on the vector modes.[3] Gauge invariance removes two of the five functions. The constraints in the action remove two others so that we are left with one degree of freedom.

In order to proceed it is convenient to work in the ADM formalism. We write the metric as

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right) \tag{2.5}$$

and the action (2.1) becomes

$$S = \frac{1}{2}\int \sqrt{h}\left[NR^{(3)} - 2NV + N^{-1}\left(E_{ij}E^{ij} - E^2\right) + N^{-1}\left(\dot{\phi} - N^i\partial_i\phi\right)^2 - Nh^{ij}\partial_i\phi\partial_j\phi\right]. \tag{2.6}$$

[3]There are no propagating vector modes for this lagrangian (2.1). They are removed by gauge invariance and the constraints. Vector modes are present when more fields are included.

Where

$$
\begin{aligned}
E_{ij} &= \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i) \\
E &= E^i_i\,. \qquad (2.7)
\end{aligned}
$$

Note that the extrinsic curvature is $K_{ij} = N^{-1}E_{ij}$. In the computations we do below it is often convenient to separate the traceless and the trace part of E_{ij}.

In the ADM formulation spatial coordinate reparametrizations are an explicit symmetry while time reparametrizations are not so obviously a symmetry. The ADM formalism is designed so that one can think of h_{ij} and ϕ as the dynamical variables and N and N^i as Lagrange multipliers. We will choose a gauge for h_{ij} and ϕ that will fix time and spatial reparametrizations. A convenient gauge is

$$
\delta\phi = 0\,, \qquad h_{ij} = e^{2\rho}[(1+2\zeta)\delta_{ij} + \gamma_{ij}]\,, \qquad \partial_i\gamma_{ij} = 0\,, \qquad \Gamma_{ii} = 0 \qquad (2.8)
$$

where ζ and γ are first order quantities. ζ and γ are the physical degrees of freedom. ζ parameterizes the scalar fluctuations and γ the tensor fluctuations. The gauge (2.8) fixes the gauge completely at nonzero momentum. In order to find the action for these degrees of freedom we just solve for N and N^i through their equations of motion and plug the result back in the action. This procedure gives the correct answer since N and N^i are Lagrange multipliers. The gauge (2.8) is very similar to Coulomb gauge in electrodynamics where we set $\partial_i A_i = 0$, solve for A_0 through its equation of motion and plug this back in the action.[4]

The equation of motion for N^i and N are the the momentum and hamiltonian constraints

$$
\begin{aligned}
\nabla_i\left[N^{-1}\left(E^i_j - \delta^i_j E\right)\right] &= 0 \\
R^{(3)} - 2V - N^{-2}\left(E_{ij}E^{ij} - E^2\right) - \dot{\phi}^2 &= 0 \qquad (2.9)
\end{aligned}
$$

where we have used that $\delta\phi = 0$ from (2.8). We can solve these equations to first order by setting $N^i = \partial_i\psi + N^i_T$ where $\partial_i N^i_T = 0$ and $N = 1 + N_1$. We find

$$
N_1 = \frac{\dot{\zeta}}{\dot{\rho}}\,, \qquad N^i_T = 0\,, \qquad \psi = -e^{-2\rho}\frac{\zeta}{\dot{\rho}} + \chi\,, \qquad \partial^2\chi = \frac{\dot{\phi}^2}{2\dot{\rho}^2}\dot{\zeta}\,. \qquad (2.10)
$$

In order to find the quadratic action for ζ we can replace (2.10) in the action and expand the action to second order. For this purpose it is not necessary to compute N or N^i to second order. The reason is that the second order term in N will be multiplying the hamiltonian constraint, $\partial L/\partial N$ evaluated to zeroth order which vanishes since the zeroth order solution obeys the equations of motion. There is a similar argument for N^i. Direct replacement in the action gives, to second order,

$$
\begin{aligned}
S = \frac{1}{2}\int & e^{\rho}(1+\zeta)\left(1+\frac{\dot{\zeta}}{\dot{\rho}}\right)\left[-4\partial^2\zeta - 2(\partial\zeta)^2 - 2V\right] + \\
& + e^{3\rho}(1+3\zeta)\left(1-\frac{\dot{\zeta}}{\dot{\rho}}\right)\left[-6\left(\dot{\rho}+\dot{\zeta}\right)^2 - \frac{2}{3}\left(\partial^2\psi\right)^2 + \dot{\phi}^2\right] \qquad (2.11)
\end{aligned}
$$

[4] As in electrodynamics in Coulomb gauge we will often find expressions which are not local in the spatial directions. In the linearized theory it is possible to define local gauge invariant observables where these non-local terms disappear.

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where we have neglected a total derivative which is linear in ψ. After integrating by parts some of the terms and using the background equations of motion (2.3) we find the final expression to second order[5]

$$S = \frac{1}{2}\int dt d^3x \frac{\dot\phi^2}{\dot\rho^2}\left[e^{3\rho}\dot\zeta^2 - e^{\rho}(\partial\zeta)^2\right]. \tag{2.12}$$

No slow roll approximation was made in deriving (2.11). Note that naively the action (2.11) contains terms of the order $\dot\zeta^2$, while the final expression contains only terms of the form $\epsilon\dot\zeta^2$, so that the action is suppressed by a slow roll parameter. The reason is that the ζ fluctuation would be a pure gauge mode in de-Sitter space and it gets a non-trivial action only to the extent that the slow roll parameter is non-zero. So the leading order terms in slow roll in (2.11) cancel leaving only the terms in (2.12). A simple argument for the dependence of (2.12) on the slow roll parameters is given below.

Since (2.12) is describing a free field we just have a collection of harmonic oscillators. More precisely we expand

$$\zeta(t,x) = \int \frac{d^3k}{(2\pi)^3}\zeta_k(t)e^{i\vec k\vec x}. \tag{2.13}$$

Each $\zeta_k(t)$ is a harmonic oscillator with time dependent mass and spring constants. The quantization is straightforward [17]. We pick two independent classical solutions $\zeta_k^{cl}(t)$ and $\zeta_k^{cl*}(t)$ of the equations of motion of (2.12)

$$\frac{\delta L}{\delta\zeta} = -\frac{d\left(e^{3\rho}\frac{\dot\phi^2}{\dot\rho^2}\dot\zeta_k\right)}{dt} - \frac{\dot\phi^2}{\dot\rho^2}e^{\rho}k^2\zeta_k = 0. \tag{2.14}$$

Then we write

$$\zeta_{\vec k}(t) = \zeta_k^{cl}(t)a_{\vec k}^\dagger + \zeta_k^{cl*}(t)a_{-\vec k} \tag{2.15}$$

where a and $a^\dagger$ are some operators. Demanding that $a^\dagger$ and a obey the standard creation and annihilation commutation relations we get a normalization condition for ζ_k^{cl}. Different choices of solutions are different choices of vacua for the scalar field. The comoving wavelength of each mode $\lambda_c \sim 1/k$ stays constant but the physical wavelength changes in time. For early times the ratio of the physical wavelength to the Hubble scale is very small and the mode feels it is in almost flat space. We can then use the WKB approximation to solve (2.14) and choose the usual vacuum in Minkowski space. When the physical wavelength is much longer than the Hubble scale

$$\lambda_{phys}H = \frac{\dot\rho e^{\rho}}{k} \gg 1 \tag{2.16}$$

the solutions of (2.14) go rapidly to a constant.

A useful example to keep in mind is that of a massless scalar field f in de-Sitter space. In that case the action is $S = \frac{1}{2}\int H^{-2}\eta^{-2}[(\partial_\eta f)^2 - (\partial f)^2]$ and the normalized classical solution, analogous to ζ_k^{cl}, corresponding to the standard Bunch Davies vacuum is [17]

$$f_k^{cl} = \frac{H}{\sqrt{2k^3}}(1-ik\eta)e^{ik\eta} \tag{2.17}$$

[5] In order to compare this to the expression in [16] set $v = -z\zeta$ in (10.73) of [16].

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where we are using conformal time which runs from $(-\infty, 0)$. Very late times correspond to small $|\eta|$ and we clearly see from (2.17) that f^{cl} goes to a constant. Any solution, including (2.17), approaches a constant at late times as $\eta^2 \sim e^{-2\rho}$, which is exponentially fast is physical time. In de-Sitter space we can easily compute the two point function for this scalar field and obtain[6]

$$\langle f_{\vec{k}}(\eta) f_{\vec{k}'}(\eta)\rangle = (2\pi)^3 \delta^3\left(\vec{k}+\vec{k}'\right) |f_k^{cl}(\eta)|^2 = (2\pi)^3 \delta^3\left(\vec{k}+\vec{k}'\right) \frac{H^2}{2k^3}\left(1+k^2\eta^2\right)$$
$$\sim (2\pi)^3 \delta^3\left(\vec{k}+\vec{k}'\right) \frac{H^2}{2k^3} \qquad \text{for } k\eta \ll 1 \tag{2.18}$$

We now go back to the inflationary computation. If one knew the classical solution to the equation (2.14) the result for the correlation function of ζ can be simply computed as

$$\langle \zeta_{\vec{k}}(t) \zeta_{\vec{k}'}(t)\rangle = (2\pi)^3 \delta^3\left(\vec{k}+\vec{k}'\right) |\zeta_k^{cl}(t)|^2 . \tag{2.19}$$

If the slow roll parameters are small when the comoving scale $\vec{k}$ crosses the horizon then it is possible to estimate the late time behavior of (2.19) by the corresponding result in de-Sitter space (2.18) with a Hubble constant that is the Hubble constant at the moment of horizon crossing. The reason is that at late times ζ is constant while at early times the field is in the vacuum and its wavefunction is accurately given by the WKB approximation. Since the action (2.12) also contains a factor of $\dot{\phi}/\dot{\rho}$ we also have to set its value to the value at horizon crossing, this factor only appears in normalizing the classical solution. In other words, near horizon crossing we set $\zeta = \frac{\dot{\phi}}{\dot{\rho}} f$ where f is a canonically normalized field in de-Sitter space. This produces the well known result

$$\langle \zeta_{\vec{k}}(t) \zeta_{\vec{k}'}(t)\rangle \sim (2\pi)^3 \delta^3\left(\vec{k}+\vec{k}'\right) \frac{1}{2k^3} \frac{\dot{\rho}_*^2}{M_{pl}^2} \frac{\dot{\rho}_*^2}{\dot{\phi}_*^2} \tag{2.20}$$

where the star means that it is evaluated at the time of horizon crossing, i.e. at time t_* such that

$$\dot{\rho}(t_*) e^{\rho(t_*)} \sim k . \tag{2.21}$$

The dependence of (2.20) on t_* leads to additional momentum dependence. It is conventional to parameterize this dependence by saying that the total correlation function has the form k^{-3+n_s} where

$$n_s = k\frac{d}{dk}\log\left(\frac{\dot{\rho}_*^4}{\dot{\phi}_*^2}\right) \sim \frac{1}{\dot{\rho}_*}\frac{d}{dt_*}\log\left(\frac{\dot{\rho}_*^4}{\dot{\phi}_*^2}\right) = -2\left(\frac{\ddot{\phi}_*}{\dot{\rho}_*\dot{\phi}_*} + \frac{\dot{\phi}_*}{\dot{\rho}_*}\right) = 2(\eta - 3\epsilon) . \tag{2.22}$$

As it has been often discussed, after horizon crossing the mode becomes classical, in the sense that the commutator $[\dot{\zeta}, \zeta] \to 0$ exponentially fast. So for measurements which only involve ζ or $\dot{\zeta}$ we can treat the mode as a classical variable.

After the end of inflation the field ϕ ceases to determine the dynamics of the universe and we eventually go over to the usual hot big bang phase. It is possible to prove [5, 16] that ζ remains constant outside the horizon as long as no entropy perturbations are

[6] In coordinate space the result for late times is $\langle f(x,t) f(x',t)\rangle sim - \frac{H^2}{(2\pi)^2}\log(|x-x'|/L)$ where is an IR cutoff which is unimportant when we compute differences in f as we do in actual experiments.

generated and a certain condition on the off-diagonal components of the spatial stress tensor is obeyed.[7] These conditions are obeyed if the universe is described by a single fluid or by a single scalar field. We should mention that for a general fluid the variable ζ can be defined in terms of the three metric as above (2.8) in the comoving gauge where $T^0_i = 0$.[8] In the case of a scalar field this implies that $\delta\phi = 0$. This gauge is convenient conceptually since the variable ζ is directly a function appearing in the metric. We see that the variable ζ tells us how much the spatial directions have expanded in the comoving gauge, so that to linear order ζ determines the curvature of the spatial slices $R^{(3)} = 4k^2\zeta$ [19]. This variable ζ is very useful in order to continue through the end of inflation since it is defined throughout the evolution and it is constant outside the horizon. An intuitive way to understand why ζ is constant is to note that the conditions stated above imply that two observers separated by some distance see the universe undergoing precisely the same history. Outside the horizon (where we can set $k = 0$ in all equations) ζ is just a rescaling of coordinates and this rescaling is a symmetry of the equations.

Other gauges can be more convenient in order to do computations in the slow roll approximation. A gauge that is particularly convenient is

$$\delta\phi \equiv \varphi(t,x)\,, \qquad h_{ij} = e^{2\rho}(\delta_{ij} + \gamma_{ij})\,, \qquad \partial_i\gamma_{ij} = 0\,, \qquad \gamma_{ii} = 0 \tag{2.23}$$

where we have denoted the small fluctuation of the scalar field by φ. In order to avoid confusion, from now on ϕ will denote the background value of the scalar field and φ will be its deviation from the background value. We expect that in this gauge the action will be approximately the action of a massless scalar field φ to leading order in slow roll. Indeed, we can check that the first order expressions for N and N^i are

$$N_{1\,\varphi} = \frac{\dot\phi}{2\dot\rho}\varphi\,, \qquad N^i_\varphi = \partial_i\chi\,, \qquad \partial^2\chi = \frac{\dot\phi^2}{2\dot\rho^2}\frac{d}{dt}\left(-\frac{\dot\rho}{\dot\phi}\varphi\right) \tag{2.24}$$

where the φ subindex reminds us that $N_{1\,\varphi}$, N^i_φ are computed in the gauge (2.23). We see that these expressions are subleading in slow roll compared to φ. So in order to compute the quadratic action to lowest order in slow roll it is enough to consider just the $(\nabla\varphi)^2$ term in the action (2.1) since V'' is also of higher order in slow roll. This is just the action of a massless scalar field in the zeroth order background. We can compute the fluctuations in φ in the slow roll approximation and we find a result similar to that of a scalar field in de-Sitter space (2.18) where the Hubble scale is evaluated at horizon crossing. After horizon crossing we can evaluate the gauge invariant quantity ζ. This is most easily done by changing the gauge to the gauge where $\varphi = 0$. This can be achieved by a time reparametrization of the form $\tilde t = t + T$ with

$$T = -\frac{\varphi}{\dot\phi} \tag{2.25}$$

[7]The condition is $\partial_i\partial_j(\delta T_{ij} - \frac{1}{3}\delta_{ij}T_{ll}) = 0$.

[8]For readers who are familiar with Bardeen's classic paper [18], we should mention that the gauge invariant definition of ζ is $\zeta = h + (\mathcal{H}^{-1}h' - A)\mathcal{H}^2/(\mathcal{H}^2 - \dot{\mathcal{H}})$ where $\mathcal{H} = \rho'$ and primes indicate derivatives with respect to conformal time and $h = H_L + H_T/3$ with A, H_L, H_T defined in [18]. In circumstances where ζ is conserved then it also reduces to the definition in terms of Bardeen potentials in [5, 16] (actually $\zeta_{here} = -\zeta_{there}$). The gauge choice that makes $h = \zeta$ is $T^0_i = 0$ or, using the equations of motion, $\dot h = \dot\rho A$.

where t is the time in the gauge (2.8) and $\tilde{t}$ is the time in (2.23). After the gauge transformation (2.25), we find that the metric in (2.23) becomes of the form in (2.8) with

$$\zeta = \dot{\rho} T = -\frac{\dot{\rho}}{\dot{\phi}} \varphi \, . \tag{2.26}$$

Incidentally, this implies that χ in (2.24) is the same as χ in (2.10). So the correlation function for ζ can be computed as the correlation function for φ times the factor in (2.26). In order to get a result as accurate as possible we should perform the gauge transformation (2.26), just after crossing the horizon so that the factor in (2.26), is evaluated at horizon crossing leading finally to (2.20). In principle we could compute ζ from φ at any time. If we were to choose to do it a long time after horizon crossing we would need to take into account that φ changes outside the horizon. This would require evaluating the action (2.1) to higher order in the slow roll parameters. Of course, the dependence for φ outside the horizon is such that it precisely cancels the time dependence of the factor in (2.26) so that ζ is constant.

In summary, the computation is technically simplest if we start with the gauge (2.23) and we compute the two point function of φ after horizon exit and at that time compute the ζ variable which then remains constant. On the other hand the computation in the gauge (2.8) is conceptually simpler since the whole computation always involves the variable of interest which is ζ. In other words, the gauge (2.23) is more useful before and during horizon crossing while the gauge (2.8) is more useful after horizon crossing.

These last few paragraphs are basically simple argument presented in [13]. The computation of fluctuations of φ in de-Sitter produces fluctuations of the order $\varphi = \frac{H}{2\pi}$ and then this leads to a delay in the evolution by $\delta t = -\varphi/\dot{\rho}$ (see (2.25)) which in turn gives an additional expansion of the universe by a factor $\zeta = \dot{\rho}\delta t = -\frac{\dot{\rho}}{\dot{\phi}}\varphi$. This additional expansion is evaluated at horizon crossing in order to minimize the error in the approximation.

We now summarize the discussion of gravitational waves [20]. Inserting (2.8) in the action and focusing on terms quadratic in γ gives

$$S = \frac{1}{8} \int \left[e^{3\rho} \dot{\gamma}_{ij} \dot{\gamma}_{ij} - e^{\rho} \partial_l \gamma_{ij} \partial_l \gamma_{ij} \right] . \tag{2.27}$$

As usual we can expand γ in plane waves with definite polarization tensors

$$\gamma_{ij} = \int \frac{d^3k}{(2\pi)^3} \sum_{s=\pm} \epsilon^s_{ij}(k) \gamma^s_{\vec{k}}(t) e^{i\vec{k}\vec{x}} \tag{2.28}$$

where $\epsilon_{ii} = k^i \epsilon_{ij} = 0$ and $\epsilon^s_{ij}(k)\epsilon^{s'}_{ij}(k) = 2\delta_{ss'}$. So we see that for each polarization mode we have essentially the equation of motion of a massless scalar field. As in our previous discussion, the solutions become constant after crossing the horizon. Computing the correlator just after horizon crossing we get

$$\left\langle \gamma^s_{\vec{k}} \gamma^{s'}_{\vec{k}'} \right\rangle = (2\pi)^3 \delta^3 \left(\vec{k} + \vec{k}' \right) \frac{1}{2k^3} \frac{2\dot{\rho}_*^2}{M_{pl}^2} \delta_{ss'} \tag{2.29}$$

where we reinstated the M_{pl} dependence. We can similarly define the tilt of the gravitational wave spectrum by saying that the correlation function scales as k^{-3+n_t} where n_t is given by

$$n_t = k\frac{d}{dk}\log\dot{\rho}_*^2 = -\frac{\dot{\phi}_*^2}{\dot{\rho}_*^2} = -2\epsilon\,. \tag{2.30}$$

3. Cubic terms in the lagrangian

In this section we compute the cubic terms in the lagrangian in two different gauges. We do this as a check of our computations. The first gauge is similar to (2.8), which is conceptually simpler since one works from the very beginning with the ζ variable in terms of which one wants to compute the answer. We need to fix the gauge to second order in small fluctuations. We achieve this by setting to zero the fluctuations in ϕ and we writing the 3-metric as

$$\begin{aligned}\delta\phi &= 0\\ h_{ij} &= e^{2\rho+2\zeta}\hat{h}_{ij}\,, \qquad \det\hat{h}=1\,, \qquad \hat{h}_{ij} = \left(\delta_{ij}+\gamma_{ij}+\frac{1}{2}\gamma_{il}\gamma_{lj}+\cdots\right)\end{aligned} \tag{3.1}$$

where $\gamma_{ii} = \partial_i\gamma_{ij} = 0$ to second order. The term proportional to γ^2 was introduced with the purpose of simplifying some formulas.[9] Note that it is necessary to define h_{ij} only to second order since any third order term in h_{ij} will not contribute to the action.

The second gauge that we choose is

$$\begin{aligned}\phi &= \phi(t)+\varphi(t,x)\\ h_{ij} &= e^{2\rho}\hat{h}_{ij}\,, \qquad \det\hat{h}=1\,, \qquad \hat{h}_{ij} = \left(\delta_{ij}+\tilde{\gamma}_{ij}+\frac{1}{2}\tilde{\gamma}_{il}\tilde{\gamma}_{lj}+\cdots\right)\end{aligned} \tag{3.2}$$

again with $\tilde{\gamma}_{ii} = \partial_i\tilde{\gamma}_{ij} = 0$. In appendix A we work out in detail the change of gauge which gives the relation between the ζ, γ_{ij} variables and the $\varphi, \tilde{\gamma}_{ij}$ variables. Here we summarize that discussion. We denote by $\tilde{t}$ the time in the gauge (3.2) and by t the time in the gauge (3.1). t and $\tilde{t}$ are related by a time reparametrization of the form $\tilde{t} = t + T(t,x)$. The function T should be such that from $\delta\phi \neq 0$ in (3.2) we get $\delta\phi = 0$ in (3.1). This determines

$$T = -\frac{\varphi}{\dot{\phi}} - \frac{1}{2}\frac{\ddot{\phi}\varphi^2}{\dot{\phi}^3} + \frac{\dot{\varphi}\varphi}{\dot{\phi}^2}\,. \tag{3.3}$$

Starting from (3.2) this time reparametrization produces a new metric. A spatial reparametrization then carries it to the form in (3.1) to second order. After all these steps we find the following relation between the φ and ζ variables (for $\tilde{\gamma} = 0$)

$$\begin{aligned}\zeta &= \dot{\rho}T + \frac{1}{2}\ddot{\rho}T^2 - \frac{1}{4}\partial_i T\partial_j T e^{-2\rho} + \frac{1}{2}\partial_i\chi\partial_i T +\\ &\quad + \frac{1}{4}e^{-2\rho}\partial^{-2}\partial_i\partial_j\left(\partial_i T\partial_j T\right) - \frac{1}{2}\partial^{-2}\partial_i\partial_j\left(\partial_i\chi\partial_j T\right)\end{aligned} \tag{3.4}$$

[9] We can define the gauge condition as $\partial_i(\log\hat{h})_{ij} = 0$.

where T is defined in (3.3) and χ is defined in (2.24). This expression will be useful for comparing results computed in different gauges. The change of variables relating $\tilde{\gamma}_{ij}$ and γ_{ij} can be found in appendix A.

3.1 Cubic term for three scalars

It is interesting to expand the action to all orders in ζ and up to cubic order in derivatives of ζ, both spatial and time derivatives. Up to total derivatives in time and space we find

$$\begin{aligned} S = \int e^{\rho+\zeta}\left(1+\frac{\dot{\zeta}}{\dot{\rho}}\right)\left[-2\partial^2\zeta-(\partial\zeta)^2\right] + e^{3\rho+3\zeta}\frac{1}{2}\frac{\dot{\phi}^2}{\dot{\rho}^2}\dot{\zeta}^2\left(1-\frac{\dot{\zeta}}{\dot{\rho}}\right) + \\ + e^{3\rho+3\zeta}\left[\frac{1}{2}\left(\partial_i\partial_j\psi\partial_i\partial_j\psi-(\partial^2\psi)^2\right)\left(1-\frac{\dot{\zeta}}{\dot{\rho}}\right)-2\partial_i\psi\partial_i\zeta\partial^2\psi\right] \end{aligned} \tag{3.5}$$

where ψ is defined in (2.10) and is of first order in ζ. Though this expression looks complicated an important point that we can easily see is that ζ is constant outside the horizon. The reason is that outside the horizon we can neglect all spatial derivatives so that only terms involving two time derivatives are left. Note there are terms involving ψ which give rise to time derivatives[10] There is no term that is important outside the horizon that contains less than two time derivatives. We can then easily see from the equations of motion that ζ becomes constant after crossing the horizon. This fact was derived in a different way in [21][11] Of course the intuitive explanation of this fact is clear, after exiting the horizon all regions evolve in the same fashion, the only difference between them is how much one has expanded relative to the other. This fact makes it clear that the definition of ζ in (3.1) [21] is the correct non-linear generalization of the variable introduced in [5].

From now on we will further expand (3.5) to cubic order in ζ. Something that is not obvious from (3.5) is that the effective cubic interaction term is of second order in slow roll. Schematically it is of the form $\epsilon^2\dot{\zeta}^2\zeta$, while the action (3.5) appears to be of order ϵ^0. Factors of ϵ are very important since they will determine whether this effect is measurable or not [8].

An easy way to see that the effective action is of order ϵ^2 is to compute the cubic term in the action in the gauge (3.2), which leads to

$$\begin{aligned} S_3 = \int e^{3\rho}\Bigg(&-\frac{\dot{\phi}}{4\dot{\rho}}\varphi\dot{\varphi}^2 - e^{-2\rho}\frac{\dot{\phi}}{4\dot{\rho}}\varphi(\partial\varphi)^2 - \dot{\varphi}\partial_i\chi\partial_i\varphi + \\ &+\frac{3\dot{\phi}^3}{8\dot{\rho}}\varphi^3 - \frac{\dot{\phi}^5}{16\dot{\rho}^3}\varphi^3 - \frac{\dot{\phi}V''}{4\dot{\rho}}\varphi^3 - \frac{V'''}{6}\varphi^3 + \frac{\dot{\phi}^3}{4\dot{\rho}^2}\varphi^2\dot{\varphi} + \frac{\dot{\phi}^2}{4\dot{\rho}}\varphi^2\partial^2\chi \\ &+\frac{\dot{\phi}}{4\dot{\rho}}\left(-\varphi\partial_i\partial_j\chi\partial_i\partial_j\chi + \varphi\partial^2\chi\partial^2\chi\right)\Bigg) \end{aligned} \tag{3.6}$$

Only the terms in the first line of (3.6) contribute to leading order in the slow roll approximation. The term proportional to V''' was considered in [2] but we see that it is subleading

[10] More precisely the term involving χ in ψ, see (2.10).

[11] In [21] it was also shown that γ is constant outside the horizon to all orders. This can also be seen in this gauge (3.1) by expanding the action to all orders in ζ and γ but second order in the derivatives. [21] call α our ζ.

in the slow roll approximation. By using the first order relation between ζ and φ (2.26) we see that the first line of (3.6) leads to an effective action which is of order ϵ^2 in the ζ variables. On the other hand, in the action (3.6), it is not obvious that there is any variable which stays constant outside the horizon. Indeed there are φ^3 couplings that typically lead to evolution of the perturbations outside the horizon [2].

Since one property (the constancy of ζ) is clear in one gauge while the other (the fact that the interaction is of order ϵ^2) is more clear in the other the reader might have some doubts about one or both statements. In order to dissipate all doubts about these statements we start from the cubic term in ζ in (3.5), do a lot of integrations by parts and drop total derivative terms to obtain

$$\begin{aligned} S_3 = & \int \frac{1}{4}\frac{\dot\phi^4}{\dot\rho^4}\left[e^{3\rho}\dot\zeta^2\zeta + e^{\rho}(\partial\zeta)^2\zeta\right] - \frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\dot\zeta\partial_i\chi\partial_i\zeta - \\ & -\frac{1}{16}\frac{\dot\phi^6}{\dot\rho^6}e^{3\rho}\dot\zeta^2\zeta + \frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\dot\zeta\zeta^2\frac{d}{dt}\left[\frac{1}{2}\frac{\ddot\phi}{\dot\phi\dot\rho} + \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}\right] + \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\partial_i\partial_j\chi\partial_i\partial_j\chi\zeta + \\ & + f(\zeta)\frac{\delta L}{\delta\zeta}\bigg|_1 \end{aligned} \tag{3.7}$$

where the first line indicates the leading order terms, which are of order ϵ^2 as expected. Note that χ is of order ϵ, see (2.10). The second line indicates higher order terms in the slow roll expansion. Finally the third line denotes terms that are proportional to the first order equations of motion (2.14). They can be removed by performing a field redefinition of the form

$$\begin{aligned} \zeta &= \zeta_n - f(\zeta_n) \\ \zeta &= \zeta_n + \frac{1}{2}\frac{\ddot\phi}{\dot\phi\dot\rho}\zeta^2 + \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}\zeta^2 + \\ & \quad + \frac{1}{\dot\rho}\dot\zeta\zeta - \frac{1}{4}\frac{e^{-2\rho}}{\dot\rho^2}(\partial\zeta)^2 + \frac{1}{4}\frac{e^{-2\rho}}{\dot\rho^2}\partial^{-2}\partial_i\partial_j(\partial_i\zeta\partial_j\zeta) + \frac{1}{2}\frac{1}{\dot\rho}\partial_i\chi\partial_i\zeta - \frac{1}{2}\frac{1}{\dot\rho}\partial^{-2}\partial_i\partial_j(\partial_i\chi\partial_j\zeta) \end{aligned} \tag{3.8}$$

where we have written the explicit expression for f.[12] After this field redefinition the action in terms of ζ_n is just the first two lines of (3.7).

By comparing the field redefinition (3.8) with (3.3), (3.4), (2.26), we find that

$$\zeta_n = -\frac{\dot\rho}{\dot\phi}\varphi \tag{3.9}$$

with no quadratic correction. This provides a consistency check on our computations since it is clear that the actions for ζ_n and φ have the same form to leading order in slow roll (the first line of (3.6) should be compared with the first line of (3.7)) and indeed the ζ and ζ_n are related as we expect by the corresponding change of gauge. The agreement between the two forms of the action should persist to all others in slow roll but we did not verify it explicitly.

[12] Note that it does not matter whether set ζ or ζ_n in the quadratic terms.

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It should be noted that the field redefinition (3.8) does indeed matter for our computation since we are interested in computing expectation values of ζ and not of ζ_n. The reason is that ζ is the variable that stays constant outside the horizon while ζ_n does not. This last fact follows from the fact that ζ is constant and the form of (3.8) where some of the coefficients of the quadratic terms are time dependent. We can also see from the action (3.7) that the second line involves a term with only one time derivative on ζ_n. This term leads to evolution of ζ_n outside the horizon consistent with what is expected from (3.8). Note that only the terms in the middle line of (3.8) are important outside the horizon.

An easy way to perform the computation is to compute using the φ or ζ_n variables up to a few Hubble times outside horizon exit of the relevant modes and then change variables to the ζ variables where it is clear that there is no evolution outside the horizon.

In order to perform the computation of the three point function we will use a variable ζ_c defined through

$$\zeta = \zeta_c + \frac{1}{2}\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}}\zeta_c^2 + \frac{1}{8}\frac{\dot{\phi}^2}{\dot{\rho}^2}\zeta_c^2 + \frac{1}{4}\frac{\dot{\phi}^2}{\dot{\rho}^2}\partial^{-2}(\zeta_c\partial^2\zeta_c) + \cdots \tag{3.10}$$

where the dots indicate terms that vanish outside the horizon or are higher order in the slow roll parameters. In terms of ζ_c the action becomes

$$S_3 = \int \frac{\dot{\phi}^4}{\dot{\rho}^4} e^{5\rho}\dot{\rho}\dot{\zeta}_c^2\partial^{-2}\dot{\zeta}_c + \ldots \tag{3.11}$$

where the dots again indicate terms of higher order in slow roll. The last term in (3.10) comes from terms proportional to the equations of motion that arise when we integrate by parts the first line in (3.7) in order to get (3.11).

3.2 Cubic term for two scalars and a graviton

We start with the computation in the gauge (3.1)

$$S = \int -2\frac{e^{\rho}}{\dot{\rho}}\gamma_{ij}\partial_i\dot{\zeta}\partial_j\zeta - e^{\rho}\gamma_{ij}\partial_i\zeta\partial_j\zeta - \frac{1}{2}e^{3\rho}\left(3\zeta - \frac{\dot{\zeta}}{\dot{\rho}}\right)\dot{\gamma}_{ij}\partial_i\partial_j\psi + \frac{1}{2}e^{3\rho}\partial_l\gamma_{ij}\partial_i\partial_j\psi\partial_l\psi\,. \tag{3.12}$$

Again, it is easiest to understand the dependence of the action on the slow roll parameter by computing the action in the gauge (3.2), where the leading contribution comes from

$$S = \int \frac{1}{2}\tilde{\gamma}_{ij}\partial_i\varphi\partial_j\varphi + \cdots \tag{3.13}$$

where the dots indicate terms that are of higher order in slow roll. We can then conclude that, despite appearances, (3.12) should be of order ϵ.

Indeed, after some integrations by parts, we find that (3.12) becomes

$$\begin{aligned} S = \int & \frac{1}{2}\frac{\dot{\phi}^2}{\dot{\rho}^2}e^{\rho}\gamma_{ij}\partial_i\zeta\partial_i\zeta + \\ & + \frac{1}{4}e^{3\rho}\partial^2\gamma_{ij}\partial_i\chi\partial_j\chi + \frac{1}{4}\frac{\dot{\phi}^2}{\dot{\rho}^2}e^{3\rho}\dot{\gamma}_{ij}\partial_i\zeta\partial_j\chi + \hat{f}(\zeta,\gamma)\left.\frac{\delta L}{\delta\zeta}\right|_1 + \hat{f}_{ij}(\zeta)\left.\frac{\delta L}{\delta\gamma_{ij}}\right|_1 \end{aligned} \tag{3.14}$$

where the first line indicates the leading order term in slow roll, which indeed has the same slow roll dependence as (3.13), once (3.9) is taken into account. The second line contains a higher order term in the slow roll approximation as well as well as terms proportional to the equations of motion. These terms can be removed by field redefinitions which we give explicitly in appendix A. These have the form that we expect from changing the gauge from (3.1) to (3.2). It turns out that these field redefinitions are not important after horizon crossing and hence are not important for our computation.

3.3 Cubic term for two gravitons and a scalar

Let us first do the computation in the gauge (3.1). By direct substitution in the action and after some integrations by parts and dropping total derivatives we get

$$\begin{aligned} S = & \int \frac{1}{16}\frac{\dot\phi^2}{\dot\rho^2}\left[e^{3\rho}\zeta\dot\gamma_{ij}\dot\gamma_{ij} + e^{\rho}\zeta\partial_l\gamma_{ij}\partial_l\gamma_{ij}\right] - \frac{1}{4}e^{3\rho}\dot\gamma_{ij}\partial_l\gamma_{ij}\partial_l\chi - \\ & -\zeta\dot\gamma_{ij}\frac{\delta L}{\delta\gamma_{ij}} + \cdots \end{aligned} \tag{3.15}$$

As usual the second line can be removed by a field redefinition. This field redefinition is the same one that we have to use to go from the gauge (3.1) to the gauge (3.2), as is discussed in more detail in appendix A. When we do the computation in (3.2) we get directly the first line of (3.15), after taking into account (3.9). It is curious that the form of the first line in (3.15) is rather similar to the first line in (3.7). As we did in that case, in order to perform the computation it is convenient to do the further field redefinition

$$\zeta = \zeta_c - \frac{1}{32}\gamma_{ij}\gamma_{ij} + \frac{1}{16}\partial^{-2}\left(\gamma_{ij}\partial^2\gamma_{ij}\right) + \cdots \tag{3.16}$$

where the dots indicate terms that vanish outside the horizon. Then the action becomes

$$S = \int \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}\dot\rho e^{5\rho}\dot\gamma_{ij}\dot\gamma_{ij}\partial^{-2}\dot\zeta_c + \cdots \tag{3.17}$$

where the dots indicates terms that are higher order in the slow roll approximation.

3.4 Cubic term for three gravitons

The computation of the term involving three gravitons is the same in the gauge (3.1) or the gauge (3.2), since after setting the scalar to zero we are changing the metric in precisely the same way. We note that the only terms in the action that contribute come from

$$S = \frac{1}{2}\int e^{2\rho}\left(\hat R + E^i_j E^j_i\right). \tag{3.18}$$

This has the same form as the result we would have obtained if we had done the computation in flat space, except for the factor of $e^{2\rho}$. The variable $\hat h_{ij}$ was defined in terms of γ in such a way that there is no cubic term involving two time derivatives. This implies that when we integrate by parts we will not need to use the equations of motion and therefore there will not be any field redefinitions. In flat space we know from the form of the scattering amplitudes that we can reduce the effective vertex to a term involving only spatial derivatives. We give its explicit form in the next section.

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4. Computation of three point functions

In this section we compute the three point functions using each of the interaction lagrangians that we found above. Before describing the computation let us make a couple of general remarks.

First let us notice that we are computing an expectation value and not a transition amplitude. We want to compute, in the interaction picture,

$$\langle \zeta^3(t) \rangle = \langle U_{\rm int}^{-1} \zeta^3(t) U_{\rm int}(t,t_0) \rangle \,, \qquad U_{\rm int} = e^{-i\int_{t_0}^{t} H_{\rm int}(t')dt'} \tag{4.1}$$

where t_0 is some early time.[13] We have suppressed the spatial dependence. To first order this gives

$$\left\langle \zeta^3(t) \right\rangle = -i \int_{t_0}^{t} dt' \left\langle [\zeta^3(t), H_{\rm int}(t')] \right\rangle . \tag{4.2}$$

For the cubic terms $H_{\rm int} = -L_{\rm int}$ after we remove all terms proportional to the equations of motion by a field redefinition.

Our second point is to note that we want to compute the expectation value in the vacuum of the interacting theory, not the vacuum of the free theory. When we do computations in Minkowski space we also have to take this into account. This can be automatically taken into account by deforming the t' integration contour so that it includes some evolution in euclidean time which projects on to the true vacuum. Fortunately in this case we can apply a similar procedure to select the vacuum. The basic reason is that at early times the physical wavelength is very small and we feel in Minkowski space, so we want the vacuum for these high energy modes to be what it is in the interacting theory in this approximately Minkowski space. In de-Sitter space this is the Hartle Hawking prescription for the vacuum [22]. In practice this will translate into a choice of contour for the integral in (4.2).[14] The evaluation of the integral in (4.2) reduces to an integration of the cubic action evaluated on the classical solutions of (2.14)[15] Since we do not have the solutions for a general potential it is useful to choose a method that minimizes the errors in the approximate evaluation of the integrals. These errors are minimized if we split the integrals in (4.2) as an integral over the region outside the horizon, the region around horizon crossing and the region deep inside the horizon. In the last region the fields oscillate rapidly and after our continuation to euclidean space there is no contribution. In the region near horizon crossing we approximate the solutions by those of de-Sitter space (2.17) and we use the action in the form that shows the leading slow roll dependence, such as in (3.11), (3.14) or (3.17). After we exit the horizon we know that ζ and γ are constant so we switch

[13] So we are not computing $\langle \zeta^3 e^{-i\int_{-\infty}^{\infty} H_{\rm int}} \rangle$ which is what we compute when we have scattering amplitudes in mind. When we compute scattering amplitudes field redefinitions do not change the answer. In our computation they do.

[14] Other choices of vacua in de-Sitter space were discussed in [23]İn inflation the admixture of these other vacua is expected to be small [24] and the leading contribution to the three point function comes from the usual vacuum.

[15] This is true only after performing field redefinitions to eliminate terms proportional to the equations of motion. Otherwise we need to take into account total derivative terms that arise when we go from (3.5) to (3.7), for example.

to those variables. This is taken into account by the field redefinitions we talked about. Then in the region well outside the horizon the fields are constant and the integral (4.2) vanishes when we do the computation in the ζ, γ variables. Below we proceed with this computation for the various cases. Due to momentum conservation there are basically two distinct kinematic configurations, the k_i can be all of the same order of magnitude of one of the k_i is much smaller than the other two. We will consider these two cases separately.

4.1 Three scalars correlator

Note that if we have a field redefinition of the schematic form $\zeta = \zeta_c + \lambda\zeta_c^2$ then the correlation function will contain two terms

$$\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle = \langle\zeta_c(x_1)\zeta_c(x_2)\zeta_c(x_3)\rangle + 2\lambda\left[\langle\zeta(x_1)\zeta(x_2)\rangle\langle\zeta(x_1)\zeta(x_3)\rangle + \text{cyclic}\right] \quad (4.3)$$

The first term is computed by (3.11). The second comes from the field redefinition (3.10). By performing different field redefinitions we can reshuffle the contributions between these two terms. Let us first compute the contribution from the action (3.11). As we explained above we evaluate this term in de-Sitter space with parameters corresponding to those of horizon exit. In de-Sitter space the contribution of an action of the form (3.11) has the form

$$\begin{aligned}\langle\zeta_c\zeta_c\zeta_c\rangle &= (2\pi)^3\delta^3\left(\sum \vec{k}_i\right)\frac{1}{\prod(2k_i^3)}\frac{\dot{\rho}_*^6}{\dot{\phi}_*^2}i\int_{-\infty}^{0} d\eta k_1^2k_2^2e^{ik_t\eta} + \text{permutations} + \text{c.c.}\\ &= (2\pi)^3\delta^3\left(\sum \vec{k}_i\right)\frac{1}{\prod(2k_i^3)}\frac{\dot{\rho}_*^6}{\dot{\phi}_*^2}4\frac{\sum_{i>j}k_i^2k_j^2}{k_t}, \qquad \text{with } k_t = k_1 + k_2 + k_3\,. \end{aligned} \quad (4.4)$$

Note that $k_i \equiv |\vec{k}_i|$. The choice of integration contour in (4.4) is such that the oscillating piece in the exponent becomes exponentially decreasing. In other words we change $\eta \to \eta + i\epsilon|\eta|$ for large $|\eta|$. This choice of contour is the one corresponding to the standard vacuum of the interacting theory.

After adding the contribution of the field redefinitions we get the final result for the three point function

$$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = (2\pi)^3\delta^3\left(\sum \vec{k}_i\right)\frac{\dot{\rho}_*^4}{\dot{\phi}_*^4}\frac{H_*^4}{M_{pl}^4}\frac{1}{\prod_i(2k_i^3)}\mathcal{A}_* \quad (4.5)$$

where the star indicates evaluation at horizon crossing and

$$\mathcal{A} = 2\frac{\ddot{\phi}_*}{\dot{\phi}_*\dot{\rho}_*}\sum_i k_i^3 + \frac{\dot{\phi}_*^2}{\dot{\rho}_*^2}\left[\frac{1}{2}\sum_i k_i^3 + \frac{1}{2}\sum_{i\neq j}k_ik_j^2 + 4\frac{\sum_{i>j}k_i^2k_j^2}{k_t}\right]. \quad (4.6)$$

In writing (4.5) and (4.6) we have assumed that all k's are of the same order of magnitude so that the moment of horizon crossing does not differ too much between the different modes. Due to momentum conservation the other possibility is that one of the k's is much smaller than the other two and these last two would be of the same order of magnitude. So we consider the configuration $k_3 \ll k_1 \sim k_2$. The mode labeled by k_3 crosses the horizon much earlier than the other modes. By the time that $k_{1,2}$ cross the horizon ζ_3 is constant. The only effect of the ζ_3 fluctuation will be to make the comoving scales $k_{1,2}$ cross the horizon at a slightly earlier time $\delta t_* = -\zeta_3/\dot{\rho}_*$. This will produce a change in the fluctuations with momenta $k_{1,2}$ due to the time dependence of the slow roll factors in (2.20). In conclusion we obtain

$$\begin{aligned}
\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\right\rangle &\sim -\left\langle \zeta_{\vec{k}_3}\zeta_{-\vec{k}_3}\right\rangle' \frac{1}{\dot{\rho}_*}\frac{d}{dt_*}\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\right\rangle \\
&\sim -n_{s*}\left\langle \zeta_{k_3}\zeta_{-\vec{k}_3}\right\rangle \left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\right\rangle \\
&\sim (2\pi)^3\delta^3\left(\sum_i \vec{k}_i\right)\frac{\dot{\rho}_*^4}{\dot{\phi}_*^2 M_{pl}^2}\frac{\dot{\rho}_{*'}^4}{\dot{\phi}_{*'}^2 M_{pl}^2}\frac{1}{2k_1^3 2k_3^3}2\left(\frac{\ddot{\phi}_*}{\dot{\rho}_*\dot{\phi}_*}+\frac{\dot{\phi}_*^2}{\dot{\rho}_*^2}\right)
\end{aligned} \tag{4.7}$$

where now * indicates the moment that k_1, k_2 cross the horizon and $*'$ indicates the time when k_3 crosses the horizon (which is earlier). In the second line we point out explicitly that this two point function involves the tilt of the scalar spectrum n_{s*} (2.22), evaluated at t_*. The prime in the first two lines of (4.7) means that we omit the factor $(2\pi)^3\delta(\vec{0})$. It can be checked that (4.5), with (4.6) goes over to (4.7) in the overlapping region of validity which is when k_3 is small but not so small to change the slow roll parameters appreciably. The first two lines in (4.7) are valid to all orders in slow roll parameters in the regime $k_3 \ll k_{1,2}$.

Our result (4.5) is of the same order of magnitude as in [3, 4], but the k dependence as well as the precise numerical coefficients are different. The reason is that [4] considers only effects due to non-linear evolution but do not consider the change in vacuum. Both of these effects are of the same order of magnitude so they should be both included and are intimately linked. Our result obeys consistency condition explained in the above paragraph while that in [4] does not, presumably because not all relevant effects were included.

Spergel and Komatsu [8, 9, 25] did an extensive analysis of the measurability of the three point function. They assumed that the three point function had the form that would follow from a field redefinition of the form[16]

$$\zeta = \zeta_g - \frac{3}{5}f_{NL}\zeta_g^2 \tag{4.8}$$

where ζ_g is gaussian. Their analysis can be roughly summarized by saying that this would be measurable if f_{NL} is bigger than around 5. This constraint comes mainly from cosmic variance if we assume that we measure the CMB up to the angular scales that the Planck satellite will measure them. See [25] for a detailed discussion of this point. Our final result (4.6) does not have the momentum dependence that would follow from (4.8). In order for that to be the case we would need that all terms in (4.6) were proportional to $\sum_i k_i^3$ which is clearly not the case. So we cannot recast our computation as a computation of f_{NL}.

[16] Spergel and Komatsu defined $\Phi = \Phi_g + f_{NL}\Phi_g^2$. The factor of 5/3 arises in the relation between the gauge invariant newtonian potential Φ and ζ during matter domination, $\zeta = -\frac{5}{3}\Phi$.

Nevertheless we can define a k-dependent f_{NL} as

$$-f_{NL} \sim \frac{5}{3}\frac{\mathcal{A}}{(4\sum_i k_i^3)} = \frac{5}{12}\left[2\frac{\ddot{\phi}_*}{\dot{\phi}_*\dot{\rho}_*} + \frac{\dot{\phi}_*^2}{\dot{\rho}_*^2}(2+f(k))\right] = -\frac{5}{12}(n_s + f(k)n_t) \tag{4.9}$$

where $f(k)$ has a range of values $0 \leq f \leq \frac{5}{6}$. $f(k)$ is a function of the shape of the triangle made by $\vec{k}_i$ and it goes to zero when two sides become much larger than the third and it becomes $5/6$ when the $\vec{k}_i$ form an equilateral triangle.

Assuming that this k dependence of the three point function does not significantly change the analysis in [8, 9, 25], we unfortunately conclude that it will not be possible to see this effect purely from the CMB. Actually, the discussion of [25] makes sense for for $f_{NL} > 1$ where we can neglect the non-linearities in the gravitational evolution after horizon reentry, some of these effects were discussed by [26, 27, 28]. In other words, to measure $f_{NL} < 1$, one has to include the leading non-linear effects in the whole evolution until we measure the temperature of the CMB.

4.2 Two scalars and a graviton correlator

This computation is rather similar to the one we did above so we will not repeat all the details. Let us note that there is no field redefinition that is important at late times so that we only need to evaluate the integral that arises from the interaction term in the first line of (3.14).

This gives

$$\left\langle \gamma^s_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\right\rangle = (2\pi)^3\delta^3\left(\sum \vec{k}_i\right)\frac{1}{\prod(2k_i^3)}\frac{\dot{\rho}_*^4}{M_{pl}^4}\frac{\dot{\rho}_*^2}{\dot{\phi}_*^2}\epsilon^s_{ij}k_2^ik_3^j 4I \tag{4.10}$$

where the transverse and traceless polarization tensor is normalized to $\epsilon^s_{ij}\epsilon^{s'}_{ij} = 2\delta_{ss'}$ and I is

$$\begin{aligned} I &= Re\left[-\int_{-\infty}^0 i\frac{d\eta}{\eta^2}(1-ik_1\eta)(1-ik_2\eta)(1-ik_3\eta)e^{ik_t\eta}\right] \\ I &= -k_t + \frac{\sum_{i>j}k_ik_j}{k_t} + \frac{k_1k_2k_3}{k_t^2}\,. \end{aligned} \tag{4.11}$$

The integral in (4.11) diverges at $\eta \to 0$ but the divergence is purely imaginary so that I is finite with our choice of contour.[17]

The dependence on the slow roll parameters is such that the three ζ correlator and the $\gamma\zeta^2$ correlator are of the same order of magnitude. After horizon reentry the amplitude of the gravitational waves decays so that for high l we still expect the three ζ correlator to dominate.

Let us now consider the correlation function in the limit $k_1 \ll k_2,\ k_3$. When k_2, k_3 are about to cross the horizon the gravity wave with momentum k_1 is already frozen so that

[17]In order to evaluate this integral it is convenient to note that $Re[-i\int_{-\infty}^0 d\eta\eta^{-2}(1-i\eta)e^{i\eta}] = -1$ with our contour prescription.

the fluctuations of ζ will be those that we expect in this deformed geometry. The main effect of the deformation is to change $k^2 \to k^2 - \gamma_{ij} k^i k^j$ in the correlation function of two ζs. This reasoning leads to

$$\begin{aligned} \left\langle \gamma^s_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \right\rangle' &\sim - \left\langle \gamma^s_{\vec{k}_1} \gamma^s_{-\vec{k}_1} \right\rangle \epsilon^s_{ij} k_2^i k_2^j \frac{\partial}{\partial k_2^2} \left\langle \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \right\rangle \\ &\sim (2\pi)^3 \delta \left(\sum \vec{k}_i \right) \frac{1}{2k_2^5} \frac{1}{2k_1^3} \frac{\dot{\rho}_*^4}{\dot{\phi}_*^2 M_{pl}^2} \frac{2\dot{\rho}_{*'}^2}{M_{pl}^2} \frac{3}{2} \epsilon^s_{ij} k_2^i k_2^j \end{aligned} \tag{4.12}$$

where * denotes the time when $k_{2,3}$ cross the horizon while *' denotes the time when k_1 crosses the horizon.

We see that (4.12) and (4.10) are consistent in the overlapping region of validity. This is a consistency check of the computation.

4.3 Two gravitons and a scalar correlator

The evaluation of this correlator using (3.16) and (3.17) is very similar to the one of the three scalar correlator. We obtain

$$\left\langle \zeta_{\vec{k}_1} \gamma^{s_2}_{\vec{k}_2} \gamma^{s_3}_{\vec{k}_3} \right\rangle = \frac{(2\pi)^3 \delta(\sum \vec{k}_i)}{\prod (2k_i)^3} \frac{\dot{\rho}^4}{M_{pl}^4} \left[-\frac{1}{4} k_1^3 + \frac{1}{2} k_1 (k_2^2 + k_3^2) + 4 \frac{k_2^2 k_3^2}{k_t} \right] \epsilon^{s_2}_{ij} \epsilon^{s_3}_{ij} \, . \tag{4.13}$$

In the case that $k_1 \ll k_{2,3}$ we also find that the correlation function is given by the derivative of the slow roll factor in the correlation function of two tensor fluctuations. We get

$$\begin{aligned} \left\langle \zeta_{\vec{k}_1} \gamma^{s_2}_{\vec{k}_2} \gamma^{s_3}_{\vec{k}_3} \right\rangle &\sim - \left\langle \zeta_{\vec{k}_1} \zeta_{-\vec{k}_1} \right\rangle' \frac{1}{\dot{\rho}_*} \frac{d}{dt_*} \left\langle \gamma^{s_2}_{\vec{k}_2} \gamma^{s_3}_{\vec{k}_3} \right\rangle \\ &\sim -n_t \left\langle \zeta_{\vec{k}_1} \zeta_{-\vec{k}_1} \right\rangle' \left\langle \gamma^{s_2}_{\vec{k}_2} \gamma^{s_3}_{\vec{k}_3} \right\rangle \\ &\sim (2\pi)^3 \delta \left(\sum \vec{k}_i \right) \frac{\dot{\phi}_*^2}{\dot{\rho}_*^2} \frac{\dot{\rho}_*^2}{M_{pl}^2} \frac{2\delta_{s_2 s_3}}{2k_2^2} \frac{\dot{\rho}_{*'}^4}{\dot{\phi}_{*'}^2 M_{pl}^2} \frac{1}{2k_1^3} \end{aligned} \tag{4.14}$$

where we have used that $\vec{k}_2 \sim \vec{k}_3$ so that $\epsilon^{s_2}_{ij} \epsilon^{s_3}_{ij} \sim 2\delta_{s_2 s_3}$. The $*$ indicates horizon crossing for k_2, k_3 and the $*'$ indicates horizon crossing for k_1 which happens earlier. In the second line we emphasized the dependence of this three point function on the tilt of the gravity wave spectrum. We see that (4.14) agrees with (4.13) in the overlapping region of validity.

4.4 Three graviton correlator

The three graviton correlator is a very similar computation. The algebra involving polarization tensors is the same as in flat space so that we can use the flat space result. We will need to do the same integral as in (4.11). The final result is

$$\left\langle \gamma^{s_1}_{\vec{k}_1} \gamma^{s_2}_{\vec{k}_2} \gamma^{s_3}_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta^3 \left(\sum \vec{k}_i \right) \frac{\dot{\rho}_*^4}{M_{pl}^4} \frac{1}{\prod_i (2k_i^3)} (-4) \left(\epsilon^{s_1}_{ii'} \epsilon^{s_2}_{jj'} \epsilon^{s_3}_{ll'} t_{ijl} t_{i'j'l'} \right) I \tag{4.15}$$

where I is given in (4.11), and t_{ijk} is given by the flat space formula (see for example [29])

$$t_{ijl} = k_2^i \delta_{jl} + k_3^j \delta_{il} + k_1^l \delta_{ij} \, . \tag{4.16}$$

We can compute this in the limit $k_1 \ll k_{2,3}$ in a way similar to what we did for the case of a graviton and two scalars

$$\left\langle \gamma^{s_1}_{\vec{k}_1} \gamma^{s_2}_{\vec{k}_2} \gamma^{s_3}_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta^3 \left(\sum \vec{k}_i \right) \frac{2\delta_{s_2 s_3} \dot{\rho}_*^2}{M_{pl}^2} \frac{1}{2k_2^5} \frac{2\dot{\rho}_{*'}^2}{M_{pl}^2} \frac{1}{2k_1^3} \frac{3}{2} \epsilon^{s_1}_{ij} k_2^i k_2^j \tag{4.17}$$

which indeed agrees with (4.15) in the overlapping region of validity.

5. Remarks on AdS/CFT and dS/CFT

5.1 AdS/CFT

The computation that we did above was done with inflation in mind, but the same mathematical structure arises if one considers a single scalar field with a negative potential. In the slow roll case, the background will be a slightly deformed anti-de-Sitter space. This can be understood as a slightly deformed conformal field theory. In other words, a non-conformal field theory which is almost conformal. An incomplete list of references where situations of this sort were considered is [30]–[35]. Here we just mention a few results that are relevant for us, for a review see [36]. The variables γ^s that we used above are associated to the traceless components of the stress tensor while the variable ζ is associated to the trace of the stress tensor. More precisely, we have a coupling of the form $\int \frac{dk^3}{(2\pi)^3} [2\zeta_{-\vec{k}} T^i_i(\vec{k}) + 2\gamma^s_{-\vec{k}} T^s(\vec{k})]$, where T^s is defined by an expression similar to (2.28), with $\gamma \to T$. The fact that the definition of the scalar mode depends on the gauge is translated into the fact that in a field theory with a scale we can either change the dimensionfull coupling constant or we can change the overall scale in the metric. It is common to fix the coupling and change the metric, which then relates ζ to the trace of the stress tensor. Alternatively we can fix the metric and change the coupling constant. In the field theory we do not have two independent operators, we have only one operator related by the equation

$$2T^i_i = \beta_\lambda \mathcal{O} \tag{5.1}$$

where β_λ is the beta function for the coupling λ which appears in the field theory lagrangian in front of the non-marginal operator as $\int \lambda \mathcal{O}$. The operator $\mathcal{O}$ is the one coupling to ϕ and the operator $2T^i_i$ couples to ζ. The factor of slow roll that relates the correlators of ζ and ϕ is precisely the factor β_λ appearing above [37].

From the computations in the previous sections we can also compute the correlation function of stress tensors and trace of the stress tensor in non-conformal theories. Depending on whether the slow roll approximation is valid or not we would need to use different formulae in those sections.

Two point functions of the trace of the stress tensor were considered in the AdS context in [33, 32, 34, 35]. The derivation of the effective action for the corresponding field in AdS identical to the one in the dS context. Similarly, computations of three point functions in AdS can be done by performing minor modifications to the above formulae. We will be more explicit below.

Now we will review the AdS_4 computation (see [38] for a review) so that we can contrast it clearly to the dS_4 computation.

Let us consider a canonically normalized scalar field in euclidean anti-Sitter space ($EAdS_4$) which is the same as hyperbolic space. The action is

$$S = R_{AdS}^2 \int \frac{dz}{z^2}\frac{1}{2}\left[(\partial_z f)^2 + (\partial f)^2\right]. \tag{5.2}$$

In order to do computations it will be necessary to consider classical solutions which go to zero for large z and obey prescribed boundary conditions at $z = z_c$. In momentum space these are

$$f_{\vec{k}} = f_{\vec{k}}^0 \frac{(1+kz)e^{-kz}}{(1+kz_c)e^{-kz_c}}, \qquad k = |\vec{k}| \tag{5.3}$$

where $f_{\vec{k}}^0$ is the boundary condition we impose at $z = z_c$. One should then compute the action for this solution as a function of the boundary conditions. Inserting (5.3) into (5.2), integrating by parts and using the equations of motion we get

$$\begin{aligned} -S &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{2}R_{AdS}^2 f_{-\vec{k}}^0 \frac{1}{z_c^2}\left.\frac{df_{\vec{k}}}{dz}\right|_{z=z_c} = -\int \frac{d^3k}{(2\pi)^3}\frac{1}{2}R_{AdS}^2 f_{-\vec{k}}^0 f_{\vec{k}}^0 \frac{k^2}{z_c(1+kz_c)} \\ &\sim -\int \frac{d^3k}{(2\pi)^3}\frac{1}{2}R_{AdS}^2 f_{-\vec{k}}^0 f_{\vec{k}}^0\left[\frac{k^2}{z_c} - k^3 + \cdots\right] \end{aligned} \tag{5.4}$$

where the dots indicate terms of higher order in z_c. The term divergent in z_c is local in position space[18] and it is viewed as a divergence in the CFT which should be subtracted by a local counterterm. The term independent of z_c is non-local and gives rise to the two point function

$$\left\langle \mathcal{O}(\vec{k})\mathcal{O}(\vec{k}')\right\rangle_{EAdS} = \left.\frac{\delta^2 Z}{\delta f_{\vec{k}}^0 \delta f_{\vec{k}'}^0}\right|_{f^0=0} \sim (2\pi)^3\delta\left(\vec{k}+\vec{k}'\right)R_{AdS}^2 k^3. \tag{5.5}$$

Where Z is the partition function of the euclidean CFT which is approximated by $Z \sim e^{-S_{cl}}$, with S in (5.4).

5.2 dS-CFT

The dS/CFT was proposed [7, 6] in analogy with AdS/CFT [39, 40, 41]. The dS/CFT postulates that the wavefunction of a universe which is asymptotically de-Sitter space can be computed in terms of a conformal field theory. More precisely, we have the formula

$$\Psi[g] = Z[g] \tag{5.6}$$

where the left hand side is the wavefunction of the universe for given three metric and the right hand side is the partition function of some dual conformal field theory. Actually the left hand side has rapidly oscillating pieces which can be expressed as local functions of the metric. We discard these pieces since they have the interpretation of local counterterms in the CFT. Here we are thinking of de-Sitter in flat slices (or Poincare coordinates) and we are imagining that all fields start in their life in the Bunch-Davies vacuum. This determines

[18] It is proportional to $\frac{1}{z_c}\int dx^3 \frac{1}{2}(\partial f^0)^2$.

the wavefunction Ψ, at least in the context of perturbation theory. If we were considering global de-Sitter space then our discussion would be valid in a small patch in the future where it can be approximated by the Poincare patch and the memory of the particular state that could have come from the far past is lost[19] This point of view follows simply from the discussion in [6] in analogy with the standard discussion in euclidean AdS where the same formula (5.6) is valid.[20] Nobody has found a concrete example of this duality and there are some suspicions that such a duality should not exist [45]. All we will do here is to do some computations on the gravity side in order to get some insight on the properties that this hypothetical CFT should have. If an example were found, then it would be a more powerful way of computing the wavefunction that semiclassical physics in de-Sitter or nearly de-Sitter space. Note that an observer living in eternal de-Sitter space will not be able to measure two point correlators such as (2.29) or the wavefunction (5.6) which involves distances much larger than the Hubble scale. Only so called "metaobservers" can measure these [6]. On the other hand if the universe is approximately de-Sitter for a while and then inflation ends and we go over to a radiation or matter dominated universe then these correlation functions become observable. In fact, we are metaobservers of the early inflationary epoch [46].

In [7, 47] the relation between CFT operators and fields in the bulk was explored and various ways of defining operators were considered. It was found that given a scalar field in the bulk one could define two operators with two conformal dimensions differing by $\Delta_+ - \Delta_- = d$ where d is the dimension of the CFT. If the field we are considering in the bulk is the metric then it is clear that the corresponding operator is the stress tensor and it should have dimension d. Indeed we will see that this agrees precisely with what we expect from the prescription (5.6). Below we explain more precisely how this computation is related both to the inflationary computation (2.29) and the corresponding EAdS computation.

The first step is to compute the wavefunction as a function of a small fluctuation in a massless scalar field f. Since f is a free field, which is a collection of harmonic oscillators, all we need to do is to compute the wavefunction for these harmonic oscillators. We want to compute the Schroedinger picture wavefunction at some time η_c as a function of the amplitude of the field f. The wavefunction is given by a sum over all paths ending with amplitude f and starting at the appropriate vacuum state. Since the action is quadratic this sum reduces to evaluating the action on the appropriate classical solution. We choose the standard euclidean (Bunch-Davies) vacuum for the fields at early times. The classical solution obeying the appropriate boundary conditions is

$$f = f^0_{\vec{k}} \frac{(1 - ik\eta) e^{ik\eta}}{(1 - ik\eta_c) e^{ik\eta_c}} \, . \tag{5.7}$$

[19]The information of the state coming from the asymptotic past in global dS is contained on modes whose angular momenta, l, on the sphere is fixed, assuming the evolution is non-singular and in the context of perturbation theory. On the other hand, we are focusing on modes with $l \gg 1$ when we look at the Poincare patch.

[20]In AdS/CFT formula (5.6) arises in the euclidean context when we think of euclidean time as the direction perpendicular to the boundary. Ψ can then be interpreted also as the Hartle-Hawking wave function [22]. See [42, 43, 44] for more on this point of view.

The boundary conditions at large η are the ones that correspond to the statement that the oscillator is in its ground state, which can be defined adiabatically at early times. The condition is that the field should behaves as $e^{ik\eta}$ for $|\eta| \to \infty$. Note that $f_{-\vec{k}} \neq f^*_{\vec{k}}$ since the boundary condition we are imposing at early times is not a real condition on the field $f(\eta, x)$.[21] This is one of the many ways to think about the harmonic oscillator wavefunction. When we evaluate the classical action on this solution we get

$$\begin{aligned} iS = i\int \frac{d^3k}{(2\pi)^3}\frac{1}{2}R^2_{dS}\frac{1}{\eta_c^2}f^0_{-\vec{k}}\partial_\eta f_{\vec{k}}|_{\eta=\eta_c} &= i\int \frac{d^3k}{(2\pi)^3}\frac{1}{2}R^2_{dS}\frac{k^2}{\eta_c(1-ik\eta_c)}f^0_{-\vec{k}}f^0_{\vec{k}} \\ \sim \int \frac{d^3k}{(2\pi)^3}\frac{1}{2}R^2_{dS}[i\frac{k^2}{\eta_c} - k^3 + \cdots]f^0_{-\vec{k}}f^0_{\vec{k}} \,. \end{aligned} \tag{5.8}$$

Note that we are dropping an oscillatory piece at $|\eta| \to \infty$ which is equivalent to slightly changing the contour of integration by $\eta \to \eta + i\epsilon$. This is the standard prescription for the vacuum state of a harmonic oscillator.

Notice that under

$$\eta = iz\,, R_{dS} = iR_{AdS} \tag{5.9}$$

the formulas (5.7) and (5.8) go into (5.3) and (5.4). The fact that (5.7) goes into (5.3) is intimately related to the statement that when the mode has short wavelength it is in the adiabatic vacuum. A consequence of this fact is that the two point function computed using dS_4 differs by a sign from the corresponding one in euclidean AdS_4.[22] More explicitly we have

$$\left\langle \mathcal{O}(\vec{k})\mathcal{O}(\vec{k}')\right\rangle_{dS_4} \equiv \left.\frac{\delta^2 Z}{\delta f^0_{\vec{k}}\delta f^0_{\vec{k}'}}\right|_{f^0=0} \sim (2\pi)^3\delta\left(\vec{k}+\vec{k}'\right)R^2_{dS}(-k^3)\,. \tag{5.10}$$

We can easily check that this is the analytically continued version of (5.5) under (5.9).

Now let us understand the relation between the wavefunction computed in (5.8), which is $\Psi \sim e^{iS_{cl}}$ and the expectation values that appeared in our earlier discussion (2.18). Of course, the relation is that $\langle f^2 \rangle = \int \mathcal{D}f f^2 |\Psi(f)|^2$. We see that only the real piece in iS contributes. This has a finite limit at late times. The divergent pieces in (5.8) are all imaginary and do not contribute to the expectation value. The functional integration over f gives again (2.18). There is a crucial factor of 2 that comes from the square of the wavefunction, so that the relation between (2.18) and (5.10) is not a Legendre transform.

Our previous discussion focused on a scalar field and its corresponding operator $\mathcal{O}$. All that we have said above translates very simply for the traceless part of the metric and the traceless part of the stress tensor, since at the linearized level the action for the graviton in the traceless transverse gauge reduces to the action of a scalar field (2.27) (2.28). We are defining the stress tensor operator as

$$T_{ij}(x) \equiv \frac{\delta Z[h]}{\sqrt{h}\delta h^{ij}(x)} = \frac{\delta\Psi[h]}{\sqrt{h}\delta h^{ij}(x)} \tag{5.11}$$

[21] There is nothing wrong in considering a complex solution since all we are doing is to evaluate a functional integral by a saddle point approximation.

[22] In other dimensions there are extra is that appears in the relation.

which is a standard definition for a euclidean field theory.[23] In this case the divergent term in (5.8) can be rewritten as $-i\frac{1}{2\eta_c}\int d^3x\sqrt{h}R^{(3)}$. Note that there is a factor of i. We want to remove this by a counterterm in the action of the euclidean CFT. These factors of i are related to the fact that the renormalization group transformation in the CFT should be appropriately unitary since this RG transformation corresponds, in the context of perturbation theory, to unitary evolution of the wavefunction in the bulk. If we define the central charge of the CFT in terms of the two point function of the stress tensor we get a negative answer. This negative answer has a simple qualitative explanation. We know that the wavefunction in terms of small fluctuations is bounded, in the sense that it is of the form $e^{-\alpha|f|^2}$ with α positive, since each mode is a harmonic oscillator with positive frequency. This sign implies a negative sign for the two point function of the stress tensor. Similarly the trace of the stress tensor is related to the derivative of the wavefunction with respect to ζ.

After we understood the relation between two point functions of operators and expectation values of the corresponding fluctuations we can similarly understand the relation between three point functions. The wavefunction has the form

$$\Psi = \mathrm{Exp}\left[\frac{1}{2}\int d^3x d^3x' \langle \mathcal{O}(x)\mathcal{O}(x')\rangle f(x)f(x') + \right.$$
$$\left. + \frac{1}{6}\int d^3x d^3x' d^3x'' \langle \mathcal{O}(x)\mathcal{O}(x')\mathcal{O}(x'')\rangle f(x)f(x')f(x'')\right] \tag{5.12}$$

where we emphasized that derivatives of Ψ give correlation functions for the corresponding operators. The expectation values in momentum space are related by

$$\begin{aligned}\langle f_{\vec{k}} f_{-\vec{k}}\rangle' &= -\frac{1}{2Re\langle \mathcal{O}_{\vec{k}}\mathcal{O}_{-\vec{k}}\rangle'} \\ \langle f_{\vec{k}_1} f_{\vec{k}_2} f_{\vec{k}_3}\rangle' &= \frac{2Re\langle \mathcal{O}_{\vec{k}_1}\mathcal{O}_{\vec{k}_2}\mathcal{O}_{\vec{k}_3}\rangle'}{\prod_i(-2Re\langle \mathcal{O}_{\vec{k}_i}\mathcal{O}_{-\vec{k}_i}\rangle')}\end{aligned} \tag{5.13}$$

where the prime means that we dropped a factor of $(2\pi)^3\delta(\sum \vec{k})$. And Re indicates the real part. The factors of two come from the fact that we are squaring the wavefunction (5.12). Notice that this explains why $\langle TT\rangle \sim c$ while $\langle \gamma\gamma\rangle \sim 1/c$ where $c \sim -R_{dS}^2 M_{pl}^2$.

Now consider three point functions. For example, consider the three point function of the traceless part of the stress tensor. This can be computed directly in dS_4 by inserting the classical solutions (5.7) into the cubic terms in the action. This gives

$$\left\langle T^{s_1}_{\vec{k}_1} T^{s_2}_{\vec{k}_2} T^{s_3}_{\vec{k}_3}\right\rangle = (2\pi)^3\delta^3\left(\sum \vec{k}_i\right)\frac{M_{pl}^2}{\dot{\rho}_*^2}\left(-\frac{1}{32}\right)\left(\epsilon^{s_1}_{ii'}\epsilon^{s_2}_{jj'}\epsilon^{s_3}_{ll'}t_{ijl}t_{i'j'l'}\right) I \tag{5.14}$$

where I is defined in (4.11). The result in $EAdS_4$ is the same as above except for a minus sign, which can be understood as coming from (5.9). When we perform this computation

[23] One might want to define it with an i so that $T_{jl} \equiv i\frac{\delta Z[h]}{\sqrt{h}\delta h^{jl}}$. This definition might be natural given that the counterterms (which represent the leading dependence of the wavefunction) are purely imaginary. In any case, it is trivial to go between both definitions.

we need to drop a local divergent term which is proportional to $\frac{-i}{2\eta_c}\sqrt{h}R^{(3)}$. We did not have any divergence in (4.17) due to the fact that we were computing the square of the wavefunction while in (5.14) we are computing the third derivative of the wavefunction. Of course, we can compute directly (5.14) from (4.17) using (5.13). So in order to compute three point functions of the stress tensor in the hypothetical three dimensional field theory corresponding to a nearly dS_4 spacetime all we need to do is apply formula (5.13) to our results in section four. To go to the corresponding expectation values in $EAdS_4$ we just need to multiply all dS_4 results by a minus sign which comes from $R_{dS}^2 \to -R_{AdS}^2$ and all correlators of the stress tensor have such a factor in front in the tree level gravity approximation.

Some of the points we explained above are specific to the four dimensional dS_4 case. The situation in dS_5 is rather interesting. The computation of fluctuations for a massless scalar field gives, outside the horizon,

$$\langle f_{\vec{k}} f_{\vec{k}'} \rangle \sim H^3 (2\pi)^4 \delta\left(\vec{k}+\vec{k}'\right) \frac{4}{\pi}\frac{1}{k^4}\,, \qquad H = R_{dS}^{-1}\,. \tag{5.15}$$

On the other hand the wavefunction $\Psi \sim e^{iS}$ has the form

$$iS = -\frac{i}{2}R_{ds}^3 \int \frac{d^4k}{(2\pi)^4} f_{\vec{k}}^0 f_{-\vec{k}}^0 \left[\frac{k^2}{2\eta_c^2} - \frac{1}{4}k^4\log(-\eta_c k) - i\frac{\pi}{8}k^4 + \alpha k^4\right] \tag{5.16}$$

where α is a real number. Note that the only term contributing to (5.15) is the real term proportional to k^4. All other terms are purely imaginary. From (5.16) we can compute the non-local contribution to the two point function which gives

$$\left\langle \mathcal{O}(\vec{k})\mathcal{O}(\vec{k}')\right\rangle_{dS_5} \sim (2\pi)^4 \delta\left(\vec{k}+\vec{k}'\right) i R_{ds}^3 \frac{1}{4}k^4 \log k\,. \tag{5.17}$$

The $EAdS_5$ answer is given by the analytic continuation (5.9). Notice that the i is due to the fact that we have an odd number of powers of R_{dS} and is consistent with the fact that the logarithmic term in the wavefunction is purely imaginary. For the stress tensor this gives an imaginary central charge and imaginary three point functions. It is rather interesting that the two point function (5.15) is related to a local term in the wavefunction, namely the term proportional to k^4, which is the only real term. In other words, the non-local piece in the wavefunction which determines the stress tensor seems unrelated to the local piece which determines the expectation value of the fluctuations. In other words, dS_5/CFT_4 would tell us how to compute the non-local piece in the wavefunction but will give us no information on the local piece. On the other hand from the inflationary point of view we would be interested in computing (5.15) which depends on the local part of the wavefunction, or the partition function of the CFT. Maybe in dS/CFT we are only allowed to use imaginary counterterms, then the field theory should be such that it allows the computation of the finite real local parts in the effective action. Note that the real term in (5.16) arises in the analytic continuation (5.9) from the term in the $EAdS_5$ wavefunction that is proportional to $k^4\log(z_c k) \to -\frac{\pi}{2}k^4 + k^4\log(-\eta_c k)$. So still, in some sense, the real part of the wavefunction (5.16) is intimately related to the non-local term in the wavefunction. It looks like this will be the situation in all odd dimensional dS spaces. The AdS_3 case studied in [7] seems special because there is no bulk propagating graviton. Stress tensor correlators in dS/CFT were also studied in [48, 47].

Now let us reexamine the three point functions of stress tensor operators in the limit that one of the momenta is much smaller than the other two. We can then approximate the small momentum by zero. This zero momentum insertion of the stress tensor can be viewed as coming from an infinitesimal coordinate transformation. So we then know that the three point function is going to be given by the change of the two point function by this coordinate transformation. For example, an insertion of the trace of the stress tensor at zero momentum is equivalent to performing a rescaling of the coordinates without rescaling the mass scale of the theory. Then the three point function will be given by the scale dependence of the two point function. In other words

$$\langle 2T^i_i(0)\mathcal{O}(k)\mathcal{O}(k')\rangle = -k^i \frac{\partial}{\partial k^i}\langle \mathcal{O}(k)\mathcal{O}(k')\rangle \tag{5.18}$$

This is the reason why three point functions in this limit are proportional to the tilt of the scalar and tensor spectra respectively, see (4.7) (4.14). There is a similar argument for the insertion of the traceless part of the stress tensor at zero momentum. Formula (5.18) is valid to all orders in slow roll.

Notice that in order to compute observable quantities from dS/CFT we will need to square the wavefunction and integrate over some range of values of the couplings and the metric of the space where the CFT is defined. In other words, in order to compute some physically interesting quantity it is not enough to consider the CFT on a fixed 3-manifold but over a range of three manifolds. This is the reason that expectation values in dS are not simply given by analytic continuation of the ones in EAdS [47] even though the wavefunction and correlation functions of operators are given by analytic continuation.[24] This makes it clear that even if dS/CFT is true there is no causality problem, one is not fixing the final state of the universe. One fixes it as an auxiliary step in order to compute the wavefunction but in order to compute probabilities we need to sum over all final boundary conditions. A slightly different integral over boundary conditions arises also in the *EAdS* context when we consider certain relevant operators [49], or double trace operators [50]. In those cases this integration is the same as a change in the boundary condition. Note that this is *not* what happens in the *dS* context since we have the *square* of the wavefunction. One might conjecture that *dS* expectation values are given by two CFTs (one for Ψ and one for Ψ^*) coupled together in some fashion. Note that then it is not clear if we should view the resulting object as a local field theory since in the resulting object is not defined on a fixed manifold since in order to compute expectation values we need to integrate over the three metric. The two copies of the CFT that we are talking about arise just at the future boundary, so these two copies are different than the two copies talked about in [6, 7, 47, 48]. In global coordinates in addition we have the past boundary. Throughout this paper we have ignored the past boundary since we focused on distances larger than the Hubble scale but smaller than the total size of the spatial slice. In the Hartle and Hawking prescription for the wavefunction of the universe the past

[24]This analytic continuation is very clear for fields with $2mR_{dS_d} < d$. For fields with mass above this bound it is not so clear what the right prescription is. In this paper we focus our attention on the easy case.

and future parts of the wavefunctions are complex conjugates of each other since the total wavefunction is real [22]. It is natural to suspect that these two pieces can be thought of as Ψ and Ψ^* in our discussion above.

Acknowledgments

I would like to thank E. Witten for many discussions and initial collaboration on these issues. I would also like to thank E. Komatsu, D. Spergel and P. Steinhardt for discussions.

This work was supported in part by DOE grant DE-FG02-90ER40542.

A. Second order change of variables between the two gauges

In this appendix we work out explicitly the change of variables to second order between the gauge (3.1) and the gauge (3.2). Let us denote by $\tilde{t}$ the time coordinate in (3.2), by t the one in (3.1). The time reparametrization is $\tilde{t} = t + T$. First let us find the first order change of variables between the two gauges. It is easy to see that we do not need to do any spatial reparametrization to first order and all we need to do is a time reparametrization. The value of T_1 at first order, as well as the relation between variables is

$$T_1 = -\frac{\varphi_1}{\dot\phi} = \frac{\zeta_1}{\dot\rho}\,, \qquad \zeta_1 = -\frac{\dot\rho}{\dot\phi}\varphi_1 \tag{A.1}$$

where the subindex reminds us that it is a first order relation.

Now we work this out to second order. In order to go from the gauge (3.2) where φ is not zero to (3.1) where φ is zero we need to do a time reparametrization determined by the equation $\phi(t + T(t)) + \varphi(t + T(t)) = \phi(t)$ which gives, to second order,

$$T = -\frac{\varphi}{\dot\phi} - \frac{1}{2}\frac{\ddot\phi\varphi^2}{\dot\phi^3} + \frac{\dot\varphi\varphi}{\dot\phi^2}\,. \tag{A.2}$$

Under this change of variables we find that the metric in (3.2) becomes

$$h^r_{ij} = e^{2\rho(t+T)}\left(\delta_{ij} + \tilde\gamma_{ij}(t) + \frac{1}{2}\tilde\gamma_{il}\tilde\gamma_{lj} + \dot{\tilde\gamma}_{ij}T + N^i_\varphi\partial_j T + N^j_\varphi\partial_i T - e^{-2\rho}\partial_i T\partial_j T\right) \tag{A.3}$$

where we have set $N = 1$ in some second order terms and N^i_φ is given in (2.24). This metric h^r_{ij} does not yet obey the gauge condition in (3.1). The violation of the gauge condition is due to some second order terms, since we already saw that at first order we do not need to do a spatial reparametrization. The terms responsible for this violation are the last four terms in (A.3). In order to make it obey those gauge condition it is necessary to do a spatial reparametrization where $\tilde{x}^i = x^i + \epsilon^i(x,t)$, where ϵ^i is of second order. The condition that ϵ^i should obey is that

$$\delta\hat{h}^r_{ij} + \partial_i\epsilon^j + \partial_j\epsilon^i = 2\alpha\delta_{ij} + \mu_{ij}\,, \qquad \partial_i\mu_{ij} = 0\,, \qquad \mu_{ii} = 0\,. \tag{A.4}$$

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Where $\delta\hat{h}^r_{ij}$ represents the last four terms in (A.3). In order to solve this equation for ϵ^i it is convenient to separate $\epsilon^i = \partial_i\epsilon + \epsilon^i_t$ where $\partial_i\epsilon^i_t = 0$. Taking the trace and $\partial_i\partial_j$ of (A.4) we obtain

$$\begin{aligned} 4\alpha &= \delta\hat{h}^r_{ii} - \partial^{-2}\partial_i\partial_j\delta\hat{h}^r_{ij} \\ &= -\partial_i T\partial_i T e^{-2\rho} + 2N^i_\varphi\partial_i T + \partial^{-2}\partial_i\partial_j(\partial_i T\partial_j T)e^{-2\rho} - \\ &\quad -2\partial^{-2}\partial_i\partial_j(N^i_\varphi\partial_j T) - \partial^2\dot{\tilde{\gamma}}_{ij}\partial_i\partial_j T \end{aligned} \tag{A.5}$$

We can similarly compute what ϵ^i are and then find μ_{ij} from (A.4). It turns out that for our purposes we can write

$$\mu_{ij} = \delta\hat{h}^r_{ij} + \mathrm{rest} \tag{A.6}$$

where the last terms are given by all other terms in (A.4). These terms vanish when integrated against a function which is traceless and divergenceless, which means that these extra terms do not contribute to our computation. In conclusion, after making this spatial reparametrization we can put the metric in a form such that it obeys the gauge (3.1) to second order.

We find then that the final field redefinition is given by

$$\begin{aligned} \zeta &= \rho(t+T) - \rho(t) + \alpha \\ \gamma_{ij} &= \tilde{\gamma}_{ij} + \dot{\tilde{\gamma}}_{ij}T + \mu_{ij} \end{aligned} \tag{A.7}$$

where T, α and μ_{ij} are given above.

It is convenient to define a variable ζ_n through the relation (3.9). Here we are thinking of ζ_n as a convenient parameterization of the variable φ. Then the explicit change of variables between the two gauges is

$$\begin{aligned} \zeta &= \zeta_n + \frac{1}{2}\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}}\zeta_n^2 + \frac{1}{4}\frac{\dot{\phi}^2}{\dot{\rho}^2}\zeta_n^2 + \\ &\quad + \frac{1}{\dot{\rho}}\dot{\zeta}_n\zeta_n - \frac{1}{4}\frac{e^{-2\rho}}{\dot{\rho}^2}(\partial\zeta_n)^2 + \frac{1}{4}\frac{e^{-2\rho}}{\dot{\rho}^2}\partial^{-2}\partial_i\partial_j(\partial_i\zeta_n\partial_j\zeta_n) + \frac{1}{2}\frac{1}{\dot{\rho}}\partial_i\chi_n\partial_i\zeta_n - \\ &\quad - \frac{1}{2}\frac{1}{\dot{\rho}}\partial^{-2}\partial_i\partial_j(\partial_i\chi_n\partial_j\zeta_n) - \frac{1}{4}\frac{1}{\dot{\rho}}\dot{\tilde{\gamma}}_{ij}\partial_i\partial_j\zeta_n \\ \gamma_{ij} &= \tilde{\gamma}_{ij} + \frac{1}{\dot{\rho}}\dot{\tilde{\gamma}}_{ij}\zeta_n - \frac{e^{-2\rho}}{\dot{\rho}^2}\partial_i\zeta_n\partial_j\zeta_n + \frac{1}{\dot{\rho}}(\partial_i\chi_n\partial_j\zeta_n + \partial_j\chi\partial_i\zeta_n) \end{aligned} \tag{A.8}$$

we see that only the first line of each field redefinition is non-vanishing outside the horizon. So fortunately we do not need to take all these terms into account in the computations in the paper. We did check however that the terms proportional to the equations of motion that arise when we integrate by parts the lagrangian in the gauge (3.1) in order to make it look more like the lagrangian in (3.2) are indeed precisely the ones that lead to the above field redefinitions.

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